Arizona Winter School 2012 Project Descriptions

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1 Patching

The aim of our first project is to extend the range of the patching techniques.

1. Try to generalize the results in Section 2 of the notes to the case when T is replaced by $k[[s,t]]$.

2. Let S be a smooth projective surface over a field k, and write F for the function field of S. Let X denote an isomorphic copy of $\mathbb{P}^1_k$ in S. For $U \subseteq X$ non-empty, let R_U denote the subring of F consisting of the rational functions on $\widehat{X}$ that are regular at the points of U. Let $\mathcal{I}$ be the ideal sheaf defining X in S, and let $\widehat{R}_U$ denote the $\mathcal{I}$-adic completion of the ring R_U. Also write $R_\varnothing$ for the subring of F consisting of the rational functions that are regular at the generic point of X, and write $\widehat{R}_\varnothing$ for its $\mathcal{I}$-adic completion. To what extent to the results of Section 2 of the notes remain true in each of the cases below?

   (a) $S = \mathbb{P}^1_k \times \mathbb{P}^1_k$, and $X = \mathbb{P}^1_k \times O$ where O is the point 0 on $\mathbb{P}^1_k$.
   (b) $S = \mathbb{P}^2_k$, and X is the line at infinity.
   (c) S is the result of blowing up the point $x = y = 0$ in $\mathbb{P}^2_k$, and X is the exceptional divisor.

   Can you make any conjectures about how the behavior depends on the choice of the pair (S, X)?

3. Let p be a prime number and consider $\mathbb{P}^1_{\mathbb{F}_p}$, with function field $\mathbb{F}_p(x)$. Can one define fields F_1, F_2, F_0 in this context, such that analogs of the results of this section hold? What if instead F is replaced by $\mathbb{Q}$? This is a very open-ended question.

2 Admissibility

Our second project is targeted at the admissibility problem.

1. With notation as in Section 5 of the notes, find explicit examples of admissible groups over F whose order is divisible by the residue characteristic of k.

2. Is every cyclic extension of $\mathbb{C}((t))(x)$ a maximal subfield of some division algebra over that field?

3. Are all cyclic groups admissible over the field of fractions $\mathbb{C}((x,y))$ of the power series ring $\mathbb{C}[[x,y]]$?

4. What can be said about admissible groups over $k((t))(x)$ if k is not algebraically closed? What if k has positive characteristic? What if $k((t))$ is replaced by $\mathbb{Q}_p$? Try to formulate conjectures.