Ex $\mathbb{G}_m$ $x+y-xy$

$$1-(1-x)(1-y)$$

over $\mathbb{Z}/p$

$$\mathrm{Aut}\,\mathbb{G}_m = \mathbb{Z}_p^\times$$

$$\lambda \in \mathbb{Z}_p^\times$$

$$x \mapsto 1-(1-x)^\lambda$$

Lubin-Tate ring $\mathbb{Z}_p$

Aut $\mathbb{G}_m$ acts trivially

$$E_0 = W[[u_1 \cdots u_{n-1}]]$$

$$E_* = W[[u_1 \cdots u_{n-1}]][u^{\pm 1}]$$

$|u| = -2$

$\mathbb{Z}_p[u^{\pm}]$ $\quad$ $\lambda \in \mathrm{Aut}(\mathbb{G}_m) = \mathbb{Z}_p^\times$

how does λ act on u?

u^{-1} = an invariant differential on $\mathbb{G}_m$

$$u^{-1} = dx + \cdots$$
$$= (1-x)\,dx$$

$$1-(1-x)^\lambda = \lambda x + \cdots$$

$$u^{-1} \mapsto \lambda \cdot u^{-1}$$

$H^{*}(\mathbb{Z}_p^{\times} : \mathbb{Z}_p[u^{\pm 1}])$ $\quad p > 2$

$\lambda^{-1} \in \mathbb{Z}_p^{\times}$ $\quad \lambda^{-1}$ - generates $\mathbb{Z}/p^{\times}$ mod p

$(\lambda^{-1})^{p-1} \neq 1$ mod p^2

$H^{1}(\mathbb{Z}_p^{\times} : u^n \mathbb{Z}_p) = \mathbb{Z}_p / (\lambda^n - 1)$

$H^0 = 0$

$n \neq 0$

makes sense for $\lambda \mapsto \lambda^n$ replaced by $\mathrm{Hom}(\mathbb{Z}_p^{\lambda}, \mathbb{Z}_p^{\times})$

$n=2$

Γ = Formal gp over k of ht 2

$\mathbb{W}[[u_1]]$

$$H^*(\mathbb{G}; \mathbb{W}[[u_1]]) = ?$$

$\uparrow$

$\operatorname{Aut}\Gamma$

It turns out $p > 3$

$$H^*(\operatorname{Aut}\Gamma; \mathbb{W}) \xrightarrow{\sim} H^*(\operatorname{Aut}\Gamma; \mathbb{W}[[u_1]])$$

$$\cong$$

$$\Lambda[x_1, x_3]$$

Dieudonne modules

k perfect field

$\mathbb{W}$ = Witt vectors of k

$k \circlearrowright \varphi \qquad \varphi(x) = x^p$

$\mathbb{W} \circlearrowright \varphi$

Dieudonné-module:

M free $\mathbb{W}$-module of finite rank

$F : \varphi^* M \longrightarrow M$

$F(am) = a^{\varphi} F(m) \qquad a \in W$

$V : M \longrightarrow \varphi^* M$

$V(a^{\varphi} m) = a V M$

$FV = VF = p$

Formal gps $/k$ $\longleftrightarrow$ Dieudonné-modules

height $\longleftrightarrow$ $\dim_W M$

dim $\longleftrightarrow$ $\dim_k M/VM$

Ex ^{2} M basis $\gamma, V\gamma$

$$F\gamma = V\gamma$$

This M $\longleftrightarrow$ height 2 formal gp $/k$

$\underline{\text{ht } n}$ $\quad \{\gamma, V\gamma, \dots V^{n-1}\gamma\}$

$$F\gamma = V^{n-1}\gamma$$

---

ht $= 2$

Aut Γ

$$\gamma \longrightarrow a\gamma + bV\gamma$$

$$V\gamma \longrightarrow a^{\varphi^{-1}}V\gamma + b^{\varphi^{-1}}V^2\gamma$$

$$= a^{\varphi^{-1}}V\gamma + pb^{\varphi^{-1}}\gamma$$

$F\gamma = V\gamma$

$p\gamma = VF\gamma$

$= V^2\gamma$

$$F\gamma \to a^{\varphi}V\gamma + pb^{\varphi}\gamma$$

$$a, b \in W\mathbb{F}_{p^2}$$

$$\mathrm{Aut}(\Gamma) = \left\{ \begin{pmatrix} a & pb^{\varphi^{-1}} \\ b & a^{\varphi^{-1}} \end{pmatrix} \middle| \begin{array}{c} a, b \\ \in \\ W\mathbb{F}_{p^2} \end{array} \right\}$$

g "

Tapis de Cartier

$$\begin{tikzcd}
M \arrow[r, dotted, "T"'] \arrow[d] & \mathbb{W} \arrow[d] \\
M/VM \arrow[r, "\simeq"'] & k \\
\gamma \arrow[r, mapsto] & 1
\end{tikzcd}$$

W - linear

Sending $\gamma \to 1$

$\Updownarrow$

lifts of n to $\mathbb{W}$.

$$u_1(T) = T(V\gamma)$$

$W[[u_1]]$

$$\gamma \xrightarrow{g} a\gamma + bV\gamma$$

$$V\gamma \longrightarrow a^{\varphi^{-1}} V\gamma + p\, b^{\varphi^{-1}} \gamma$$

$$g(u_1) = \frac{a^{\varphi^{-1}} u_1 + p\, b^{\varphi^{-1}}}{b u_1 + a}$$

Crystalline "approximation"

$$M \longrightarrow E_{-2} = uW[[u_1, \ldots u_{n-1}]]$$

$$\gamma \longrightarrow u$$

$$V\gamma \longrightarrow uu_1$$

$$\vdots$$

$$V^{n-1}\gamma \longrightarrow uu_{n-1}$$

Aut Γ equivariant

$$G \qquad G \simeq G_a$$

$$\begin{array}{ccc} W & \longrightarrow & G \otimes W \\ \downarrow & & \\ k & & \end{array}$$

Γ

$$f(x) = \log_G(x) = x + \cdots \in G \otimes W[[x]]$$

$$f^{-1}(f(x) + f(y)) = x +_G y$$

<u>Ex</u> $\log_{G_a}(x) = \sum \frac{x^n}{n}$

$T : M \to W$

$$f(x) = \sum_i T(F^n \gamma) \frac{x^{p^n}}{p^n}$$

is the log of a formal gp over W.

<u>Ex</u> $G = \mathbb{G}_m$

$M = \cancel{\{\gamma\}}\ W \cdot \{\gamma\}$

$$F\gamma = \gamma$$

$$\log = \sum \frac{x^{p^n}}{p^n}$$

<u>Ex</u> $ht = 2$ $\qquad T(\gamma) = 1$

$\qquad T(V\gamma) = 0$

$$f(x) = \sum \frac{x^{p^{2n}}}{p^n} = \ell(x)$$

$\cancel{W[[w]]}$ $W[[w_1]]$ $\qquad T(\gamma) = 1$

$\qquad T(V\gamma) = w_1$

$$f(x) = \ell(x) + \frac{w_1}{p}\,\ell(x^p)$$

$f^{-1}(f(x) + f(y))$ does not have coefficients in

$$W((w_1)).$$

It does have coeff in the divided power 'completion

$$W\langle\langle w_1 \rangle\rangle$$